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19 June 2020

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June 19, 2020

A revised draft is forthcoming in *Studies in Nonlinear Dynamics and Econometrics*

# Causal Relationships between Inflation and Inflation Uncertainty


**Abstract**

Since the publication of Friedman's (1977) Nobel lecture, the relationships between the mean function of the inflation stochastic process and its uncertainty, and between inflation uncertainty (IU) and real output growth have been the subject of much research, with some studies justifying this causality and some reaching the opposite conclusion or finding an inverse correlation between mean inflation and inflation volatility with causation in either direction. We conduct a systematic econometric study of the relationships between the first two moments of the inflation stochastic process and between IU and output growth using state-of-the-art approaches and propose a time-varying inflation uncertainty measure based on stochastic volatility to consider unpredictable shocks. Further, we extend the literature by providing a new econometric specification of this relationship using two semi-parametric approaches: the frequency evolutionary co-spectral approach and continuous wavelet methodology. We theoretically justify their use through an extension of Ball's (1992) model. These frequency approaches have two advantages: they provide the analyses for different frequency horizons and do not impose restriction on the data. While the literature focused on the US data, our study explores these relationships for five major developed and emerging countries/regions (the US, the UK, the euro area, South Africa, and China) over the past five decades to investigate the robustness of our inferences and sources of inconsistencies among prior studies. This selection of countries permits investigation of the inflation versus inflation uncertainty relationship under different hypotheses, including explicit versus implicit inflation targets, conventional versus unconventional monetary policy, independent versus dependent central banks, and calm versus crisis periods. Our findings show a significant relationship between inflation and inflation uncertainty, which varies over time and frequency, and offer an improved comprehension of this ambiguous relationship. The relationship is positive in the short and medium terms during stable periods, confirming the Friedman–Ball theory, and negative during crisis periods. Additionally, our analysis identifies the phases of leading and lagging inflation uncertainty. Our general approach nests within it the earlier approaches, permitting explanation of the prior appearances of ambiguity in the relationship and identifies the conditions associated with the various outcomes.



**Keywords:** Inflation, Inflation uncertainty, Output growth, Frequency approach, Wavelet, Semi-parametric approach, Stochastic volatility.

**JEL classification: C14, E31.**

## 1. Introduction

Uncertainty, which often refers to unpredictable volatility (Grier and Perry, 1998), is an important economic theory concept, as it could affect consumers' saving, investors' and policymakers' decisions, economic well-being, and the entire economy (Rossi et al., 2016). Particularly, uncertainty about future inflation, which is considered among the most important inflation costs, is a main concern to monetary policymakers. According to Cukierman and Meltzer (1986) and Evans and Wachtel (1993), inflation uncertainty (IU) can occur through at least two main sources. First, significant differences among international monetary policy regimes could lead to IU, similar to conventional versus unconventional monetary policies. Second, IU could also be induced by policy regime uncertainty.[1] Furthermore, as economic agents often use new information to update their perceptions regarding the actions of central banks, IU would be time varying and potentially complex to measure.

Therefore, the relationship between inflation and IU has been the focus of several theoretical and empirical studies to investigate the latter's effect. For instance, Friedman (1977) found a positive relationship between inflation and IU, suggesting that inflation causes IU. Using a game with asymmetric information and two policymakers, consisting of a liberal policymaker prepared to disinflate and bear the economic cost of reducing inflation and a conservative policymaker not prepared to do so, Ball (1992) developed a formal justification for Friedman's theory. This relationship is known as the Friedman–Ball relationship or theory.[2] This positive relationship between inflation and IU can also be found in Logue and Willet (1976) and Fischer and Modigliani (1978). However, recent related empirical studies focused

[1] Policymakers could use information unavailable to the public or create an inflation surprise, but the public might not know the weight assigned to this surprise creation. Furthermore, policymakers can sometimes find ambiguous procedures useful.
[2] The principle is that when inflation is low (as it was in the US in the 1960s), there is the consensus that policymakers will strive to keep it low. However, if inflation is high (as in the late 1970s), there will be a dilemma for policymakers. Either they disinflate, which can produce a recession risk, or they do not. The public will then be unclear about the intentions of policymakers. Even if disinflation occurs, its timing would be uncertain, increasing IU anyway.

on US inflation and indicated some ambiguity in the relationship between inflation and IU (see, e.g. Golub, 1994).

Therefore, the objectives of our research are analogous to those of Barnett et al. (1997), who investigated the source of nonrobustness in time series inferences about nonlinearity, and of Barnett and Chen (2015), who investigated the nonrobustness of policy inferences from structural macroeconometric models. However, the approaches to resolving the nonrobustness problems differ as follows. Barnett et al. (1997) ran a controlled competition among competing time series tests, while Barnett and Chen (2015) investigated the bifurcation stratification of the structural parameter spaces. In this paper, we the extend inflation–IU testing to time-frequency analysis to include prior approaches into our more general econometric analysis.

This study revisits the Friedman hypothesis. First, we revisit the relationship between inflation and IU to investigate its ambiguity for some developed and emerging countries. Second, we complete the previous studies through analyzing the IU-real output growth relationship. We conduct these investigations in a systematic econometric manner using state-of-the-art methodology. To date, the importance of the relationship has been motivated in macroeconomics in various ways. First, the co-existence of targeted and non-targeted monetary policies yields heterogeneous effects on price stability, inflation policy, and subsequently, on IU, which impact the economic activity. This can be illustrated by the variety in the levels and volatility of inflation among countries. Second, the fact that some central banks are more independent and have larger mandates than others has several implications for inflation and the relationship between inflation and IU. The more independent the central bank is, the more we expect an increase in uncertainty to imply an inflation fall, and vice versa. Third, the recent global financial crisis has directly affected liquidity and prompted some monetary authorities to switch from conventional to unconventional monetary policies, which could increase uncertainty and influence how IU impacts inflation, and economic in general.[3] Finally, the differences in monetary policy mandates produce diverse time horizons for countries to achieve the monetary policy goals.

Those horizons can be characterized as short term (less than one year), medium term (from one to three years), and long term (more than three years). Consequently, we suggest *a priori* that the relationship between inflation and IU as well as IU-output growth could vary across horizons. We address this concern by using evolutionary econometric analysis to assess the time-varying effect of IU. This is a central focus of our analysis.

Our study makes three fundamental contributions to this field as follows. We propose a time-varying estimate of IU using the stochastic volatility model of Berument et al. (2009) and Ferreira and Palma (2016) to consider unexpected shocks. Unlike previous studies that often apply parametric econometric models, our study uses two non-parametric approaches, the evolutionary co-spectral approach and wavelet methodology, to reproduce the time-varying relationship between inflation and IU. This new specification of the inflation and IU relationship with non-parametric models can be theoretically justified by our extension of Ball's (1992) model. Finally, we complete the previous studies through examining the IU-output growth relationship across different time-horizons.

Our methodological choice has many advantages.[4] First, nonparametric approaches do not require data restrictions or pre-treatment,  unlike time-series models. Second, these approaches enable us to investigate the relationship over continuous time to yield a time-varying analysis of the inflation–IU relationship and IU-output growth and a decomposition of these relationships across different horizons. Third, the wavelet approach transforms these time series into different frequency components, thus providing an alternative representation of variable

[3] Mallick and Mohsin (2016) show that this switching induces inflation shocks that might affect the real economy and particularly the consumption of durable rather over non-durable ones.
[4] To the best of our knowledge, this is the first study that assesses the relationship between inflation and a time-varying measure of IU using multivariate frequency approaches.

variability on a scale-by-scale basis. The result presents a more informative analysis of the inflation–IU and IU-output growth relationships, while identifying, for example, the periods of lead and lag uncertainty. Our results are particularly useful in resolving the ambiguity associated with this relationship.

We focus on a heterogeneous sample of five major developed and emerging countries/regions: the US, the UK, the euro area, China, and South Africa. As our objective is to produce conclusions that are robust across difference circumstances, these choices include countries/regions with explicit and implicit inflation targets. Further, central bank commitment varies across countries, enabling us to test the effects of central bank independence on how uncertainty affects inflation and vice versa as well as the IU-output growth. Some of these countries still rely on conventional monetary policies (e.g., China), while others use unconventional monetary policies. This study is the first to focus on the effects of the latter on the inflation–IU and IU-output growth relationships. The extension of the sample period to the aftermath of the global financial crisis enables us to verify this relationship during both calm and crisis periods. The sample period varies by country, according to data availability, but we include data both before and after the recent global financial crisis for all countries.

Overall, our sample enables us to test the effects of different hypotheses on inflation and uncertainty. In the aftermath of the recent global financial crisis, there has been a tendency towards deflation in several countries, while uncertainty remains relevant. This sample should help us determine whether the inflation–IU and IU-output growth relationships has changed over the past few years.

Our analysis yields three main results. First, we note a significant relationship between inflation and IU, which varies with time and frequency. Specifically, this relationship alternates between being positive in the short and medium terms during stable periods, confirming the Friedman–Ball theory, while becoming negative during crisis periods. Second, our results distinguish between the periods where IU is leading from those where it is lagging. This relationship also varies per country, suggesting significant effects of the monetary policy regime, target objective, and degree of central bank independence on uncertainty. Finally, the double specification of this relationship of time and frequency and our theoretical justification provide new findings that could be helpful to predict the uncertainty effects and drivers more reliably. Further, our results cast light on the ambiguity associated with this relationship in previous studies (e.g., Golub, 1993).

The remainder of this paper is organized as follows. Section 2 briefly discusses the theoretical framework and literature on the inflation–IU relationship. Section 3 presents the econometric methodology. The main empirical results are discussed in Section 4. Section 5 concludes the paper.

## 2. Theory and Related Literature

### 2.1 Theoretical Background

The relationship between inflation and IU has been investigated in the literature from both directions, each drawing upon different economic backgrounds. Table 1 shows each background is divided into two strands, based on the sign of the relationship between inflation and IU.

The first economic background stipulates that inflation causes IU and is disaggregated into two strands based on the sign of this causality direction relationship. Friedman (1977) argued that the relationship between inflation and IU is positive. Indeed, in his Nobel Prize lecture entitled "Inflation and Unemployment," he stated that the monetary policy objective of boosting employment would increase inflation. Theoretically, the central bank aims to confront inflationary pressure but, in practice, Friedman explained that monetary authorities could behave differently. Consequently, the public would be uncertain about the future policy, leading to more IU.

Under the same background, Ball (1992) provided a more formal justification of Friedman's hypothesis. He used an asymmetric information game between the Federal Reserve and the public. The game postulates two types of policymakers: liberal and conservative. The public knows that a conservative policymaker is willing to bear the economic costs of reducing inflation, while the liberal policymaker is not. However, the public is not certain about who will be the future policymaker. If inflation is low, uncertainty will be low as well, since either policymaker will maintain inflation low. However, during high inflation periods, only the conservative policymaker will disinflate. Therefore, when inflation is high, doubt about the identity of the future policymaker will cause IU also to be high. Based on Ball's (1992) model, Friedman's hypothesis is correct: increased inflation would raise IU.

Pourgerami and Maskus (1987) also agreed with Friedman's on the direction of the relationship between inflation and IU, but argued that the sign of the relationship is negative. Under a high inflation rate, agents tend to forecast the inflation rate more accurately, since they invest more resources to avoid forecasting errors, thus having significant related costs. Accordingly, a high inflation rate will decrease IU.

**Table 1: Inflation–IU Relationship Theories**

| **Sign of relationship** / **Causality of relationship** | **(+)** | **(-)** |
|---|---|---|
| **Inflation causes IU** | Friedman (1977)<br>Ball (1992) | Pourgerami and Maskus (1987)<br>Ungar and Zilberfarb (1993) |
| **IU causes inflation** | Cukierman and Meltzer (1986) | Holland (1995) |

The second economic background claims that the causality relationship is the inverse, with IU causing inflation. Under this tradition, there also are two contradictory analyses, differing regarding the sign of the causality relationship between uncertainty and inflation. For instance, Cukierman and Meltzer (1986) applied a game theoretic model to central banks. Their framework is based on the Barro–Gordon model of Federal Reserve behavior, emphasizing money supply uncertainty and the objective function of policymakers. The model concludes that an increase in uncertainty implies a corresponding increase in inflation, as policymakers seek to create an inflation surprise to stimulate output growth. Then, uncertainty positively causes inflation. Conversely, Holland (1995) found that uncertainty can have a negative impact on inflation. The independent central bank decreases inflation following an IU increase, to reduce the real cost of IU.

Overall, this relationship between inflation and IU has been the subject of complexity and ambiguity in either background.[5] To resolve this ambiguity and understand the specificities of the relationship, we reconsider the first background above, that is, the hypothesis that inflation causes IU. Particularly, we extend Ball's (1992) model to better characterize the shock properties in his model. The proposed extended model allows our time-frequency analysis of this relationship to explain and resolve its ambiguous character.

More formally, Ball's (1992) model identifies two types of policymakers: conservative policymakers (C), who focus only on inflation, and liberal policymakers (L), who focus on unemployment as well as inflation. Their loss functions over period $t$ are:

$$\begin{cases} L_t^C = a\,\pi_t^2 \\ L_t^L = a\,\pi_t^2 + (U_t - U_t^*), \end{cases} \tag{1}$$

where $L_t^C$ and $L_t^L$ are the loss functions of the conservative and liberal policymakers, respectively; $U_t$ and $U_t^*$ represent actual and optimal social unemployment, respectively; and $\pi_t$ represents the inflation rate at period $t$. The optimal social unemployment is assumed to be time-invariant.

According to the short-run Phillips curve, unemployment is determined as follows:

$$U_t = U^N - (\pi_t - \pi_t^e), \tag{2}$$

[5] On the subject of complex dynamics, see Barnett et al. (2015).

where $\pi_t^e$ is the expected inflation rate at time $t$, given the available information at period $t$ - 1. The natural rate of unemployment is $U^N$.

To avoid the time inconsistency problem, the natural unemployment rate is defined as $U^N = U^* + 1$.[6] Therefore, by combining (1) and (2), the liberal policymaker's loss function is written as follows:

$$L_t^L = a\,\pi_t^2 + (\pi_t - \pi_t^e - 1). \tag{3}$$

In this model, policymakers are assumed to alternate their power stochastically, based on a Markov process. Specifically, Ball (1992) assumed that, in period $t$, L are in power. They are characterized by a probability $p$, at which they would lose power and would be replaced by C in the following period, $t + 1$, and vice versa.

Further, as per Canzoneri (1985) and Ball (1992), we assume that the policymakers in power define their objective through the following inflation target $(\pi_t^*)$:

$$\pi_t = \pi_t^* + \delta_t, \tag{4}$$

where $\delta_t$ presents a stochastic shock.

Equation (4) stipulates that the two policymakers separately define an expected inflation rate $(\pi_t^e)$, which is assumed to be rational. Then, the policymakers would individually define their inflation targets $(\pi_t^*)$, which depend on the inflation level in the previous period, $t$ – 1.

In practice, the equilibrium of the policymakers is defined after fixing the inflation target by minimizing the expected present value of their respective loss functions. That is, based on this game, we have several cases that differ on the policymaker in power and the function of the inflation level at $t$ – 1. These cases are summarized in Table 2.

[6] In line with previous studies (Barro and Gordon, 1983), time inconsistency is defined to occur when the natural rate of unemployment is higher than optimal social unemployment, leading to an inflation bias.

**Table 2: Conservative versus Liberal Policymakers**

| ***Case 1: C are in power at t* - 1** | ***Case 2: L are in power at t* - 1** |
|---|---|
| If C are in power at $t$ - 1, their inflation expectation for period $t$ will be zero, as their monetary objective is to only reduce inflation. In addition, L expect zero inflation for period $t$ since, at $t$ – 1, C are in power. | If L are in power at $t$ - 1, their inflation expectation for period $t$ depends on the previous inflation level. They are tempted to increase employment by a positive inflation. However, their temptation is deterred by their adverse attitude to inflation bias, resulting from temporal inconstancy. Therefore, to avoid such problems, Ball (1992) assumed they would be tempted toward a positive inflation, but below a threshold level ($\bar{\pi}$).<br>***<u>Case 2.1: If $\pi_{t-1} < \bar{\pi}$ (low inflation):</u>***<br>L target an inflation rate equal to zero. This behavior is explained by the fact that L are tempted to increase their target but fear to exceed threshold $\bar{\pi}$. In case of C, they continue to target zero inflation.<br>***<u>Case 2.2: If $\pi_{t-1} > \bar{\pi}$ (high inflation):</u>***<br>L target an inflation rate $\pi_t^{*+} > \bar{\pi}$. Behind their preference, L avoid an aggressive reaction to reduce inflation, as they fear recession. However, C target an inflation rate equal to zero, as their main objective is to reduce inflation.<br>As the probability of L still being in power is 1 - p, the expected inflation will be $(1-\text{p})\ \pi_t^{*+}$. |

Based on the above model, economic shocks will define the level of inflation at period $t$ (Equation (4)). According to the above cases, the expected inflation rate could take two possible values: zero or $(1-\text{p})\ \pi_t^{*+}$. When $\pi_t^e = E[\pi_t|\omega_{t-1}] = 0$, there is no uncertainty. However, when $\pi_t^e = E[\pi_t|\omega_{t-1}] = (1-\text{p})\ \pi_t^{*+}$, based on Equation (4), the inflation level during period $t$ will have the following variance:

$$Var[\pi_t] = Var\ [\pi_t^* + \delta_t] = Var[\delta_t]. \qquad (5)$$

Equations (1)–(5) summarize Ball's (1992) model. To motivate our use of time-scale approaches, which can explore the relationship between inflation both over time and over frequency, we extend Ball's model. Specifically, we relax the assumption of homogenous inflation shocks ($\delta_t$) in Ball's model in favor to two shock types: short-memory shocks

($e^S$) might be presented through an ARMA (AutoRegressive Moving Average) process and long-memory shocks ($e^L$) on an ARFIMA (Fractional ARMA) process. Therefore, the inflation shock is assumed to be the sum of short and long terms shocks. The inflation shock is as follows:

$$\delta_t = e_t^L + e_t^s, \tag{6}$$

where $e_t^L$ represents a long-memory shock, associated with a further change in monetary policy, which might affect inflation in the long term (Evans, 1991), while $e_t^s$ is a short-memory shock in the money demand, affecting inflation in the short term (Ball, 1992).[7]

Let us now reconsider Equation (5) on the inflation variance, while considering our shock decomposition in Equation (6). Accordingly, the IU, $Var[\,\pi_t]$, will depend on the nature of the shocks: short-term versus long-term. If there is only a short-term shock and the monetary policy is unchanged, the IU could be specified as

$$Var[\,\pi_t] = Var\,[\pi_t^* + \delta_t] = Var[\delta_t] = var\,[e_t^s] \;=\; \sigma_S^2 + p(1-p)\pi_t^{*+}. \tag{7}$$

However, if there is a long-term shock, which affects the economy, the IU could be expressed as follows:

$$Var[\,\pi_t] = Var\,[\pi_t^* + \delta_t] = Var[\delta_t] = var\,[e_t^L] = \sigma_L^2 + p(1-p)\pi_t^{*+}. \tag{8}$$

The above reconsideration of IU dynamics in Ball's (1992) model through the hypothesis on shock nature suggests that IU dynamics might differ and exhibits further asymmetry and complexity, according to the type and horizon of the shock affecting the economy. Additionally, IU's interaction with inflation might differ with the horizon and the type of inflation shock. Interestingly, breaking the IU (Equation (6)) into short- and long-memory shocks motivates the choice of the time-frequency approach to investigate the relationship between inflation and IU. Specifically, econometric literature (e.g., Jensen, 1999, 2004) highlighted the robustness of the wavelet approach in identifying long-memory behaviors in economic time series. Therefore,

[7] Hereafter, we can empirically show that the interest of using the wavelet and frequency approaches to understand the inflation–IU relationship for different time-scales and frequencies can be justified by the presence of different short- and long-term shocks.

for a better characterization of this inflation–IU relationship, we investigate it for different time horizons and scales by using a time-frequency approach. This empirical approach will be further discussed after reviewing the related literature.

### 2.2 Related Literature

Most of previous studies analyzing the Friedman hypothesis have focused on the inflation-IU relationship. These studies have focused on the inflation–IU relationship, but there is still no consensus about either the direction or sign of this relationship.[8] For instance, using the autoregressive conditional heteroscedasticity (ARCH) and generalized ARCH (GARCH) models, Ben Nasr et al. (2015) concluded that the average US inflation is not related to uncertainty. Baillie et al. (1996) applied a fractionally integrated GARCH model and also found no relationship between inflation and uncertainty in the US. However, they found significant relationships in the UK, Brazil, Argentina, and Israel. Grier and Perry (1998) investigated the inflation–IU relationship for G7 countries during 1948–1993. While their causality analysis supports the Friedman–Ball hypothesis that inflation Granger causes uncertainty, they obtain mixed results on the causality effect from uncertainty to inflation. Further, their results vary among countries and the authors have mentioned that the response to IU might be correlated with central bank independence. Indeed, for Japan and France, the relationship is consistent with Cukierman and Meltzer (1986), in which increased uncertainty is related to higher inflation, while for the US, Germany, and the UK, the result is the opposite.

Kontonikas (2005) studied the relationship between IU and inflation in the UK during 1972–2002, and found a positive correlation between past inflation and current uncertainty. Additionally, the author showed that adopting an explicit inflation target by the Bank of England reduces inflation persistence and uncertainty. Moreover, by applying parametric models of long memory to the US, the UK, and Japan during 1962–2001, Conrad and Karanasos

[8] We intentionally mention only recent, important, and related studies; see Ben Nasr et al. (2015) for a more complete literature review. Golub (1993) also provides a concise analysis and survey of this complex relationship.

(2005) validated the Friedman hypothesis that inflation increases IU. They also found that IU affects inflation in Japan and the UK differently. Using a stochastic volatility in mean model, Berument et al. (2009) studied the effects of IU on inflation in the US during 1976–2006. The authors found an increase in inflation followed a positive shock affecting inflation volatility. That result is in line with Cukierman and Meltzer (1986) and Cukierman (1992).

Neanidis and Savva (2011) investigated the relationship between nominal uncertainty and inflation in the European Union (EU), and found that uncertainty positively affects inflation in the pre-EU access period. Further, they indicated the absence of any effect during EU access and entry. Using a Markov-regime switching asymmetric GARCH-in-mean model, Chang (2012) investigated the inflation–IU relationship under the hypotheses of regime switching and non-normality in the US during 1960–2011. The author showed that IU does not affect inflation, while inflation affects IU negatively during periods of high-inflation volatility, but not periods of low-inflation volatility. The main advantage of Chang's (2012) study is relaxing the restrictions on the distribution of random errors and the use of a switching-regime framework. Zapodeanu et al. (2014) focused on Romania to show a significant bilateral-causality relationship between inflation and IU. Mallick and Sousa (2013) focused on the BRICS and found that important commodity price shocks might lead to an increase in inflation, requiring aggressive action from central banks towards inflation stabilization, thus yielding IU.

Finally, Ben Nasr et al. (2015) also applied a Markov-switching vector autoregressive model to investigate the relationship between inflation and IU in South Africa during 1921–2012. Their findings did not reject Friedman's hypothesis, but the causality relationship was found to be valid only when unidirectional. Creal and Wu (2014) developed a new macro-financial model to study the effects of interest rate uncertainty on business cycles. They showed that a shock onto the short interest rate negatively affects inflation, while a higher, long-term uncertainty shock has a positive effect. A related literature strand investigates the relationship between money supply growth uncertainty and the economy. See, for example, Serletis and Rahman (2009a, b), Serletis and Shahmoradi (2006), and Serletis and Xu (2017).

Overall, previous studies do not provide unanimous conclusions about the inflation–IU relationship, and their findings vary with the methodologies, countries, and samples under consideration. However, all previous studies, except Albulescu et al. (2019) as shown in Table 3, tested this relationship using parametric models, which imply restrictions on specifying the relationship between inflation and uncertainty. Additionally, the sign and size of the IU effect on inflation depends on the *a priori* level of the central bank's commitment to inflation. Indeed, with weak commitment, a positive effect can be expected, while a negative effect is anticipated if the central bank has a strong commitment. Moreover, the effect of inflation on uncertainty might differ depending on the conduct of explicit or implicit inflation targets. For instance, Johnson (2002) showed that, while a formal target has a negative effect on inflation, it significantly impacts uncertainty. Finally, most of previous studies investigated the Friedman hypothesis from one side regarding the relationship between inflation and uncertainty.

All these different empirical findings confirm Golub's (1993) findings, who showed that both survey strategy, based on surveys of economists and consumers, and forecasting model strategy, based on restricted-uncertainty (GARCH) models of IU across exchange rate regimes, yield mixed findings and ambiguity regarding the inflation–IU relationship.[9] To reconcile this disagreement, he considered a downtrend in uncertainty and showed that the non-consideration of such effects in previous studies may have biased their results.

We propose to resolve this ambiguity and the inconsistent results on the inflation–IU relationship in previous research by proposing an alternative, more general strategy. Interestingly, we aim to complete the gap of previous studies, as they analyzed the Friedman

[9] Particularly for Golub (1993), survey analyses confirm the Friedman–Ball hypothesis, while forecasting approaches are less conclusive, as restricted-uncertainty models provide mixed results, and the exchange rate model identified no relationship between inflation and IU.

hypothesis through one side related to Inflation-IU relationship as shown in Table 3. This study fills this gap through investigating both inflation-IU and IU-real output growth relationships. Contrary to the dominant literature strand using variants of the GARCH model in modeling IU, we present a time-varying, latent measure of IU based on stochastic volatility, in line with Berument et al. (2009), Chan and Grant (2016), and Ftiti and Jawadi (2018). Moreover, we adopt a time-frequency approach that includes prior approaches within our more general approach.

**Insert Table 3.**

# 3. Econometric Methodology

## 3.1 Inflation Uncertainty Measure

Before analyzing the relationship between inflation and IU, we need to provide a measure for IU. The earlier literature employed the standard deviation or the variance of inflation as proxies for inflation uncertainty. However, these measures only capture inflation variability and not inflation uncertainty (which depends on the variations of that nonconstant variance). Evans (1991) emphasizes that uncertainty should not be treated as variability. For example, observing low volatility does not imply low uncertainty as economic agents might still have little information about inflation and, therefore, consider the future as highly uncertain. Following this, ARCH and GARCH models (e.g., Emery, 1993; Holland, 1993) and their variants (Kontonikas, 2005; Ben Nasr 2015; among others) were employed to estimate conditional variance of inflation as a proxy for inflation uncertainty. These have recently been criticized. Therefore, other studies (Berument et al., 2009; Ferreira and Palma, 2016) modeled conditional variance as an unobserved component based on a Markov process, also known as stochastic volatility (SV) models. In addition to the time-varying behavior of SV models, they are considered more flexible than previous classical measures by embodying two separate disturbance terms (Carnero et al., 2004). Furthermore, the latent specification in these models rejects any ad-hoc assumptions on the specification of conditional volatility. Additionally, the SV proxy is more appropriate to capture the unpredictable characteristics of uncertainty.

In practice, Chan and Grant (2016) recently compared different variants of GARCH and SV models and confirmed the superiority of the latter. Although this type of measure is mostly adopted for financial time series analysis (Gourieroux and Sufana, 2010; Koopman et al., 2010), some recent studies developed stochastic volatility measures for economic time series, such as inflation (Chan, 2015) and exchange rate (Chan and Hsiao, 2014). Interestingly, in the literature on inflation and IU, the SV model has been used as a measure of IU by Berument et al. (2009), Chan (2015), Ferreira et Palma (2016), and Ftiti and Jawadi (2018), but with heterogeneous results.

We retain stochastic volatility as a proxy of IU. Specifically, we apply a stochastic volatility model based on moving average student-t errors developed by Chan (2013).[10] To estimate the stochastic IU, we adopt the efficient sampler method proposed by Chan (2013) and specified as follows.

The inflation series is represented as:

$$\pi_t = \mu_t + \tau_t, \tag{9}$$

$$\tau_t = \epsilon_t + \psi\, \epsilon_{t-1}, \tag{10}$$

where $\epsilon_t \sim N(0, \exp(h_t))\ for\ t = 1, \dots, T$. However, the state is assumed to evolve into a stationary condition based on the following equation:

$$h_t = \mu_h + \phi_h(h_{t-1} - \mu_h) + \zeta_t, \tag{11}$$

where $\zeta_t \sim N(0, \sigma_h^2)\ for\ t = 1, \dots, T$, with $\zeta_t$ and $\epsilon_t$ being independent for all leads and lags. The stationarity condition of $(h_t)$ is $|\phi_h| < 1$. The states are initialized with $h_1 \sim$

[10] There are also other methods of modeling of stochastic volatility that are less interesting, such as Gaussian errors models or heavy tails and serial dependence.

$N\left(\mu_h, \frac{\sigma_h^2}{1-\phi_h^2}\right)$.

This specification is completed by independent prior distributions for $\mu_h$, $\phi_h$, and $\sigma_h^2$, so that

$$\mu_h \sim N(\mu_{h_0}, V_{\mu_h})\ ;\ \ \phi_h \sim N\left(\phi_{h_0}, V_{\phi_h}\right) I(|\phi_h| < 1)\ ;\text{ and } \sigma_h^2 \sim IG(v_h, S_h),$$

where $I[*]$ is an indicator function and IG the inverse-gamma distribution.

The conditional variance of the inflation series ($\pi_t$) is time-varying based on two channels. First, the moving average component based on previous variance ($\epsilon_{t-1,}$) in Equation (10) and from the log-volatility ($h_t$) evolves according a stationary AR(1) process (Equation (11)). Our stochastic volatility model is appropriate for modeling inflation dynamics for developed economies. Indeed, it is characterized by a high level of persistence, fractionally integrated, and long-memory dynamics (Jensen, 2009). Therefore, our approach considers all these potential characteristics of inflation series.

### 3.2 Time-Scale Approaches

Unlike previous studies, we use two time-scale approaches to investigate the relationship between inflation and IU: the evolutionary co-spectral approach and the wavelet method. These approaches have at least two advantages over time-series models. First, they provide analysis at multiple frequencies. Consequently, the analysis of this relationship is explored over the short, medium, and long terms. Second, these approaches are non-parametric, thus requiring no hypotheses on the distributions and no parameter estimation.

In this study, we used both frequency approaches, at least for two reasons. First, both approaches are complementary. Indeed, the spectral approach is used for discrete time and therefore it gives us the coherence (relationship) between inflation-IU for specific time-scale. The wavelet approach is based on continuous transform, gives the relationship between inflation and IU in continuous times, which offers us a continuous measure of the causality between the studied series. Second, employing both approaches aims to check the robustness of the frequency framework in analyzing the relationship between IU-inflation.

### 3.2.1 Evolutionary Co-Spectral Density Function

Priestley (1965) extended the spectral approach of stationary processes to the non-stationary case by proposing the evolutionary spectral approach. Here, the word "evolutionary" refers to a time-dependent time series $X(t)$. This approach has been recently used by several authors, such as Allégret and Essaadi (2011), Ftiti (2010), Van Bellegem (2013), and Van Bellegem and Von Sachs (2008). Priestley and Tong (1973) extended the analysis to the bivariate case to study the relationship between two processes associated with an inflation series based on coherence measure $C_{t,XY}^{2}(w)$. We denote those two series by *X*(*t*), which is inflation, and *Y*(*t*), which represents IU. The coherence function is interpreted as a linear relationship between the corresponding components of time series $X(t)$ and $Y(t)$. We note that this measure is equivalent to correlation in the time-domain approach, except that the signal is squared for our coherence measure. Therefore, the coherence measure ranges from 0 to 1, while the classical correlation ranges from -1 to 1.

Coherence is defined as:

$$C_{t,XY}^{2}(w) = \frac{{h^2}_{t,XY}}{h_{t,XX}h_{t,YY}}, \tag{12}$$

where $h_{t,XX}$ *and* $h_{t,YY}$ are the estimated auto-spectral density functions for inflation, *X*(*t*), and IU, *Y*(*t*), respectively. The cross-spectral density function between the two processes is $h_{t,XY}$.[11]

Priestley (1965, 1966) defined the suitable windows for estimations of the spectral and co-spectral density functions as:

[11] For more details on the estimation of auto- and cross-spectral density functions, see Ftiti (2010).

$$g(u)=\begin{cases}\frac{1}{2\sqrt{h\,\pi}} & if\ \ |u|\preccurlyeq h\\ 0 & if\ \ |u|>h\end{cases}\qquad W_v=\begin{cases}\frac{1}{T'} & if\ \ |v|\preccurlyeq\frac{T'}{2}\\ 0 & if\ |v|>\frac{T'}{2},\end{cases}\tag{13}$$

where $\preccurlyeq$ means less than or equal to but close to, and with small probability possibly greater than. We adopt the same window parameters used by Artis et al. (1992), $h=7$ and $T'=20$. This choice of values is consistent with conditions (*i*) and (*ii*) below and provides robust estimators. With $\hat{h}_k(t,w)$ being the estimate of the spectral density, $h_k(t,w)$, Priestley (1988) concluded that $\hat{h}_k(t,w)\approx h_k(t,w)$, while var $\hat{h}_k(t,w)$ decreases when $T'$ increases, and $cov\left(\hat{h}\,(t_1,w_1),\hat{h}\,(t_2,w_2)\right)=0,\ \ \forall(t_1,t_2),\forall(w_1,w_2)$, if at least one of the following conditions is satisfied: $(i)$ $|t_1-t_2|\ \geq T'$ and $(ii)$ $|w_1\pm w_2|\geq\frac{\pi}{h}$. To respect conditions $(i)$ and $(ii)$, we choose $\{t_i\}$ and $\{w_j\}$ so that:

$$t_i=\{18+20i\}_{i=1}^{I},\tag{14}$$

where $I=\left[\frac{T}{20}\right]$ with $T$ being the sample size, and

$$w_j=\left\{\frac{\pi}{20}(1+3(j-1)\right\}_{j=1}^{7}.\tag{15}$$

To consider condition (*ii)*, the following frequencies can be retained: $\frac{\pi}{20},\frac{4\pi}{20},\frac{7\pi}{20},\frac{10\pi}{20},\frac{13\pi}{20},\frac{16\pi}{20},\frac{19\pi}{20}$. However, we focus on only three frequencies in carrying out analyses over the short, medium, and long terms. In practice, the shift from the frequency to the time domain is based on ratio $\frac{2\pi}{\lambda}$, where $\lambda$ is the frequency. The long-run coherence function for three years and three months is based on frequency $\frac{\pi}{20}$, the middle-run of approximately one year refers to $\frac{4\pi}{20}$, and the two-month short-run coherence is defined by the $\frac{19\pi}{20}$.[12]

[12] The selected frequencies are also confirmed by the spectral function between inflation and IU, having the most energy among other frequencies. Due to space considerations, we did not report all figures of the spectral for our sample. However, these figures are available upon request from the authors.

### 3.2.2. Wavelet Approach: Theory and Estimation

We choose a second frequency approach through wavelets, which useful for non-stationary time-series analyses. The wavelet approach enables investigating the relationship between two non-stationary time series through its continuously resized window properties.[13]

There are different wavelet groups that can be used in analyzing time series, such as discrete versus continuous and real versus complex. The continuous wavelet has often been used in previous studies (e.g. Gallegati et al., 2014; Haven et al., 2012; Madaleno and Pinho, 2014; Rua and Nunes, 2012), as it is the most helpful with the time and scale resolution of time-series decomposition and helps overcome the limitations of the other types of wavelets. Although the discrete wavelet has an orthogonal time-scale presentation, the continuous wavelet is more appropriate for several reasons. First, it avoids any data length constraint to ensure the decomposition of time series as in the case of the discrete wavelet. Second, its properties that may be more suitable to noise than those of other decomposition techniques (Aguiar-Conraria and Soares, 2011). Finally, under the continuous wavelet, it is more useful to identify the point of time in which a variable leads or lags, as understanding the exact underlying lead-lag phenomenon between variables is often difficult for policy analysts (Tiwari, 2013).

We choose the Morlet wavelet to obtain a better balance between time and scale resolutions. The Morlet wavelet was first introduced by Goupillaud et al. (1984) and can be expresses as follows:

$$\psi_{\Theta}(\mu) = \pi^{\frac{-1}{4}} e^{i w_a \mu} e^{\frac{-1}{2}\mu^2}, \tag{16}$$

where: $w_0$ and $\mu$ are defined as dimensionless frequency and time scales, respectively.

For the Morlet wavelet, the central frequency ($w_0$) equals six, which is considered a

[13] For low frequencies, window width is high and low for high frequencies. Consequently, a signal with a large window suggests coarse features, while a small window suggests fine features.

good choice to ensure a relevant balance between time and scale resolutions (see Grinsted et al., 2004). For this central frequency, $w_0 = 6$, the Fourier period $\lambda_{wt}$ is almost equal to the scale $\left(\lambda_{wt} = \frac{w_0}{2\pi} scale = \frac{6}{2\pi} scale \approx 1\, scale\right)$. Additionally, the relationship between the scale, $s$, and frequency, $f$, is given by $f \approx \frac{1}{s}$. The wavelet is drawn out over time by varying its scale, $s$, normalized to have unit energy and defined as $s = \frac{\mu}{t}$. For a discrete time series $x_n$ $\{n = 1 \ldots, N\}$ of $N$ observations with a uniform time step $(\Phi t)$, the continuous wavelet transform is given by:

$$W_n^x(s) = \sqrt{\frac{\Phi t}{s}}\ \sum_{m=1}^{N} X_n\, \psi_0 \left[(m-n)\frac{\Phi t}{s}\right], \tag{17}$$

where $\Phi t$ is the time step.

The wavelet power spectrum for a time series, $x_n$, with $N$ observations is defined as $|W_n^x(s)|^2$ and represents the local variance of time series, $x_n$. Therefore, after defining the continuous wavelet transforms for each time-series analysis, $x_t$ (inflation) and $y_t$ (IU), we define the cross-wavelet transform. The measure of this cross-wavelet spectrum that captures the covariance between the two time series, $x(t)$ and $y(t)$, in the time-frequency space, $W_x$ and $W_y$, is defined as:

$$W_n^{xy}(s) = W_n^x(s) W_n^y\,(s), \tag{18}$$

where $W_n^x(s)$ and $W_n^y\,(s)$ are the wavelet transforms for time series $x_t$ and $y_t$, respectively. The cross-wavelet power is defined by $\left|W_n^{xy}(s)\right|^2$and interpreted as the local covariance between the two time series.[14]

In the modeling of the causality relationship over the time domain, we define the phase

[14] Specifically, the cross-wavelet power between inflation and its uncertainty measures the similarity of the powers in these series. The statistical significance level of the cross-wavelet power was defined by Torrence and Compo (1998),

difference, $\varphi_{x,y}$, between inflation, $x_t$, and its uncertainty, $y_t$, as the tool providing information on the delays in the oscillation between inflation, $x_t$, and its uncertainty, $y_t$ (Bloomfield et al., 2004). The phase difference depicts the relative position of the pseudo-cycle inflation and its uncertainty according to Equation (19):

$$\varphi_{x,y}(s) = tan^{-1}\left(\frac{\mathcal{J}[W_n^{xy}(s)]}{\mathcal{R}[W_n^{xy}(s)]}\right), \tag{19}$$

where, $\mathcal{J}$ and $\mathcal{R}$ denote the imaginary and real parts of the cross-wavelet, respectively.

To analyze the phase difference between inflation and its uncertainty, we note it ranges between $[-\pi, \pi]$. If $\varphi_{x,y}(s) = 0$, inflation and its uncertainty move together, a phase analogous to positive covariance. When $\varphi_{x,y}(s) \in \left[-\frac{\pi}{2}, 0\right]$, inflation and its uncertainty are in phase and inflation is leading. When $\varphi_{x,y}(s) \in \left[0, \frac{\pi}{2}\right]$, inflation and its uncertainty are in phase and IU is leading. For $\varphi_{x,y}(s) \in \left[\frac{\pi}{2}, \pi\right]$, inflation and its uncertainty are in anti-phase and inflation is leading. For $\varphi_{x,y}(s) \in \left[-\pi, -\frac{\pi}{2}\right]$, inflation and its uncertainty are in anti-phase and IU is leading.[15]

Further, to analyze the relationship between inflation and its uncertainty, we adopt wavelet coherence, as defined by Torrence and Webster (1999). This measure is associated with the coherence function in Equation (20) and the dynamic correlation in the conventional time series. However, the wavelet coherence function is superior to other measures, as it identifies both the causality effect and lead-lag phase phenomena between two time series. The wavelet coherence function is defined as

[15] We note that, in the coherence or cross-wavelet spectrum graphics, it is not easy to obtain a phase according to these different ranges. Therefore, the lead-lag relationship is reproduced through arrows pointing in different directions in the circular mean. This circular mean provides the significance of the phase lead–lag relationship. To determine the phase between the two series, we must estimate the mean and confidence interval of the phase difference in line with Grinsted et al. (2004, pp. 4–5), who used the circular mean defined by Zar (1999).

$$\mathrm{R}_n^2(s) = \frac{\left|\varepsilon\left(s^{-1}W_n^{xy}(s)\right)\right|^2}{\varepsilon|(s^{-1}|W_n^{x}(s)|^2)|.\varepsilon\left|\left(s^{-1}\left|W_n^{y}(s)\right|^2\right)\right|}, \tag{20}$$

where $\varepsilon$ is a smoothing operator.[16] When squared, our coherence function ranges from 0 to 1, unlike a classical correlation measure, which would range from -1 to 1. The statistical significance of the coherence function is estimated through the Monte Carlo method, in accordance with Torrence and Compo (1998) and Grinsted et al. (2004).

# 4. Data and Empirical Analysis

## 4.1 Data and Preliminary Analysis

The data include the consumer price indexes (CPI) for three developed regions, the US, UK, and euro area, and two major emerging countries, China and South Africa. The CPI is required to compute the inflation rate from Equation (21), while the IU is computed using the stochastic volatility of inflation:

$$inf_{i,t} = 100 * \mathrm{Ln}\left(\frac{CPI_{i,t}}{CPI_{i,t-1}}\right), \tag{21}$$

where index $i$ represents the region and the t the month.

Monthly data are obtained from Datastream and selected for each country, depending on data availability: January 1999–March 2015, January 1988–March 2015, January 1950–March 2015, January 1960–December 2012, and January 1986–March 2015 for the euro area, the UK, the US, South Africa, and China, respectively.

These countries show important changes in their monetary policy conduct over these periods, including explicit versus implicit inflation targets and unconventional versus conventional monetary policies. The US is the best-documented case, with at least four important phases in its conduct of monetary policy: policy oriented toward unemployment in

[16] For more details, see Torrence and Webster (1999).

the 1970s, policy focused more on money after the 1979 oil shocks, policy focused on exchange rate and financial stability after the 1980s, and the unconventional monetary policies since 2008.[17] On January 25, 2012, The Fed adopted an explicit inflation target of 2%, but nevertheless continued to follow a monetary policy with two main objectives, price stability and economic growth. Furthermore, while the US monetary policy is characterized by emphasis on rules, it has a high degree of discretion, thus increasing uncertainty about inflation. The UK adopted an explicit inflation target in 1992. Further, in May 1997, the Bank of England acquired operational independence in setting its short-term interest rate. There are some similarities between the monetary histories of the US and the UK, particularly regarding their conduct of monetary policies and inflation targeting (see, e.g., Conrad and Karanasos, 2005). For the euro area, the European Central Bank (ECB) adopted an implicit inflation target policy of around 2%, and this policy covers several EU countries that did not apply an inflation target before 1999. However, the ECB has only one stated objective, price stability. On the other hand, South Africa adopted an informal inflation target in 1990 and a formal one in 2000 ranging between 3% and 6%. Finally, China's monetary policy aims to maintain currency stability and improve economic growth. China does not follow inflation targeting.

Accordingly, unlike previous studies, the inclusion of countries with different target strategies, distinct degrees of independence, and heterogeneous monetary policies permit a comparative analysis of the inflation–IU relationship across countries.

We began by performing unit root tests to determine whether all inflation series are I(0). The results are consistent with the inflation rate dynamics in the studies mentioned in Table 1. [18] We report in Table 4 the main descriptive statistics.

[17] In the US, inflation increased due to the increase in defense spending in mid-1965 because of the Vietnam war, after the first oil price shock in 1973, after the elimination of price and wage controls in 1974, and after the second oil shock during 1979–1980. See Bernanke and Mishkin (1992) for details.

[18] We do not report the results of unit root tests due to space considerations but these results are available upon request.

**Table 4. Descriptive Statistics of Inflation Rate and Normality Test**

| | Mean (%) | Min (%) | Max (%) | Standard deviation | Kurtosis | Skewness | Jarque-Bera test |
|---|---|---|---|---|---|---|---|
| UK | 0.220 | -0.618 | 2.338 | 0.238 | 22.16 | 2.463 | (0.00) |
| US | 0.290 | -1.735 | 1.846 | 0.343 | 5.555 | 0.171 | (0.00) |
| Euro area | 0.154 | -0.449 | 0.693 | 0.170 | 4.573 | -0.553 | (0.00) |
| China | -0.012 | -3.711 | 3.364 | 0.851 | 5.624 | -0.195 | (0.00) |
| South Africa | 0.672 | -0.742 | 4.118 | 0.553 | 7.013 | 1.212 | (0.00) |

Note: Values in (.) denote the p-values of the Jarque-Bera test.

Table 4 displays a lower level of inflation with a higher standard deviation for China than for the other regions. This volatility excess can be explained by the fact that, among these countries, only China does not pursue price stability. China has various other monetary objectives, including currency value stability and promotion of economic growth, with the use of several monetary instruments, including reserve requirement ratio, interest rate, rediscounting, lending, and open market operations. Kurtosis statistics are positive for all regions' inflation rates, with the highest value for the UK, suggesting fat tail behavior. The inflation rate distributions for the UK, the US, and South Africa are right skewed, while those for the euro area and China are left skewed. This leptokurtic excess and asymmetry are inconsistent with normality, according to the Jarque-Bera test, and suggests that inflation might react differently to being shocked positively or negatively.

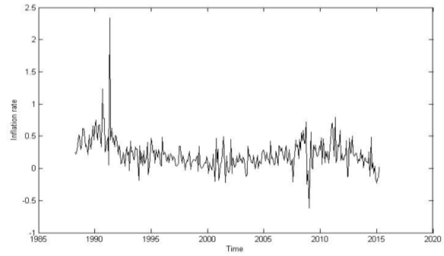
### 4.2 Modeling the Dynamics of Inflation Uncertainty

Figure 1 reports the dynamics of inflation rates and Figure 2 plots the IU based on the stochastic volatility measure described in subsection 3.1. We report in Table 5 the main descriptive statistics of the IU measure.[19]

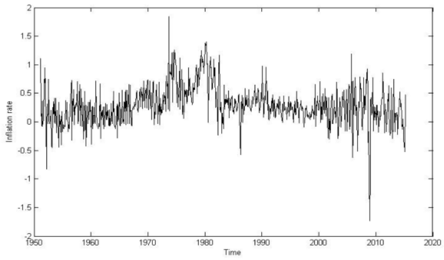
[19] We did not report the results of the estimations coefficients of stochastic volatility models to due to space considerations. However, these results are available upon request.

## Figure 1. Inflation rates for all sample countries

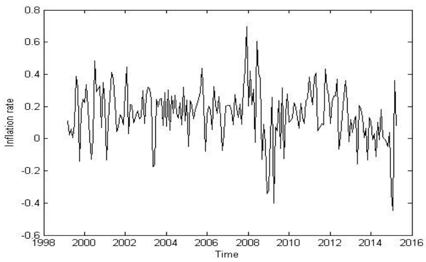
1(a) UK



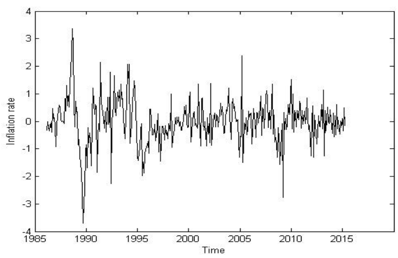

1(b) US



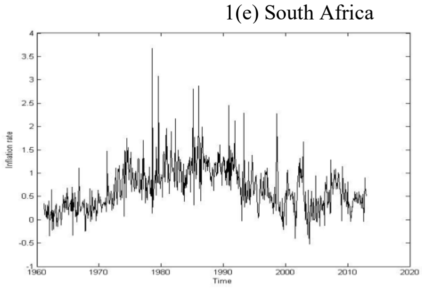

1(c) Euro area




1(d) China

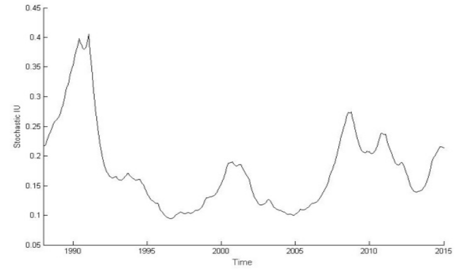



1(e) South Africa

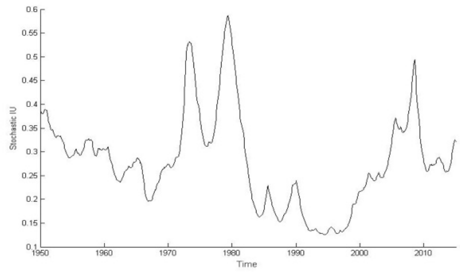



## Figure 2. IU for all sample countries

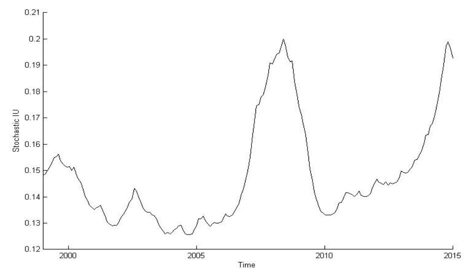
2(a) UK



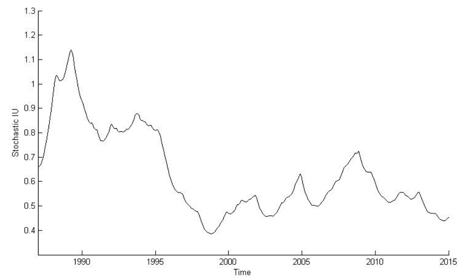

(b) US



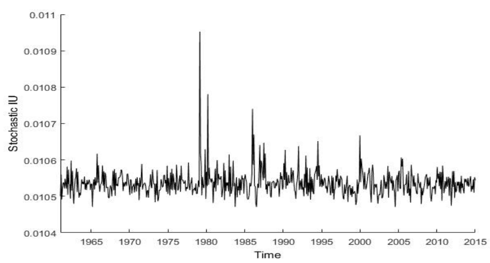

2(c) Euro area




2(d) China




3(e) South Africa



The following results are evident from Table 5. China has the highest level of IU, although it has the lowest inflation rate on average. Additionally, China has the most volatile inflation and IU. This suggests that the absence of an explicit inflation target could be the source of an increase in uncertainty and volatility. Kurtosis statistics are positive for all IUs, providing further evidence of fat tails. Further, all IU series are right skewed, justifying the non-Gaussian distribution of the conditional inflation volatility. The Jarque-Bera test significantly rejects normality for all series. Overall, these preliminary, indirect tests might suggest that IU could appear asymmetrically and nonlinearly with inflation and, thus, exhibit a time-varying behavior.

**Table 5. Summary Statistics IU Measure**

| | Mean | Min | Max | Standard deviation | Kurtosis | Skewness | Jarque-Bera test |
|---|---|---|---|---|---|---|---|
| UK | 0.174 | 0.092 | 0.368 | 0.065 | 3.725 | 1.050 | (0.000) |
| US | 0.279 | 0.125 | 0.595 | 0.100 | 3.503 | 0.727 | (0.000) |
| Euro area | 0.153 | 0.126 | 0.217 | 0.025 | 3.125 | 1.113 | (0.000) |
| China | 0.617 | 0.419 | 0.898 | 0.126 | 2.104 | 0.575 | (0.000) |
| South Africa | 0.435 | 0.291 | 0.708 | 0.095 | 2.861 | 0.618 | (0.000) |

Note: Values in (.) refer to the p-values of the Jarque-Bera statistic test.

To investigate inflation and IU properties directly and specify their relationship, we first test the causality hypothesis between inflation and IU in a linear framework using the Granger linear causality test. Next, we model this relationship in a nonlinear context with non-parametric econometric tests using a double time-frequency approach.

### 4.3 Modeling the Relationship between Inflation and IU with Parametric Tests

The comparison of the inflation curve and IU dynamics in Figures 1 and 2 provides further evidence of the similarities between the two measures. For example, in the UK, when inflation increased at the beginning of the 1990s, the IU simultaneously experienced a peak, suggesting a positive relationship. However, in the aftermath of the global financial crisis, the relationship seems to be the opposite, as inflation decreases but uncertainty increases, reflecting the sign change in the relationship.

We also compute the non-conditional correlation matrix, apply the Granger causality test, and perform comparative analyses to analyze the inflation–IU relationship (see Table 6). The correlation is around 50% for the UK and South Africa, but does not exceed 33% for the U.S., which is consistent with Friedman's theory. However, the correlation does not exist for China and is negative for the euro area. The equality mean test significantly rejects the null hypothesis for the UK and South Africa. To better investigate this relationship, we conduct Granger causality testing and find evidence of bilateral causality relationships. However, given the rejection of the normal distribution, we have to carefully analyze the results of these parametric tests. To better understand the relationship between inflation and IU, we next apply two non-parametric approaches: evolutionary co-spectral analysis and the wavelet approach.

**Table 6. Relationship between Inflation and IU**

| | UK | US | Euro area | China | South Africa |
|---|---|---|---|---|---|
| **Non-conditional correlation** | 0.492 | 0.335 | -0.242 | 0.0174 | 0.485 |
| **Granger causality test** | | | | | |
| Inflation does not Granger cause IU | 44.750*** | 2.521* | 3.653** | 3.389** | 77.497*** |
| | (0.000) | (0.081) | (0.027) | (0.035) | (0.000) |
| IU does not Granger cause inflation | 18.666*** | 9.121*** | 2.555* | 0.619 | 29.858*** |
| | (0.000) | (0.000) | (0.080) | (0.538) | (0.000) |
| **Equality mean test** (p-value) | 0.001 | 0.398 | 0.980 | 0.800 | 0.000 |

**Note:** Values in (.) denote the p-values for different tests. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

## 4.4. Modeling the Relationship between Inflation and IU with Nonparametric Tests

### 4.4.1. Evolutionary Co-Spectral Analysis

Figures 3–5 report the dynamic interaction between inflation and IU for the long, medium, and short terms, respectively, where the long run is defined as more than three years, medium run one year, and short run three months. Overall, these figures point to several conclusions. The relationship between inflation and uncertainty exhibits a significant time variation, which confirms the dynamic relationship between inflation and IU, as noted in previous empirical studies. However, our specification captures the most important stylized facts associated with inflation increases. Indeed, the coherence function reaches high levels during periods of crisis and shock, including the oil shocks, dot-com bubble, and recent global financial crisis, reflecting peak uncertainty during market downturns.

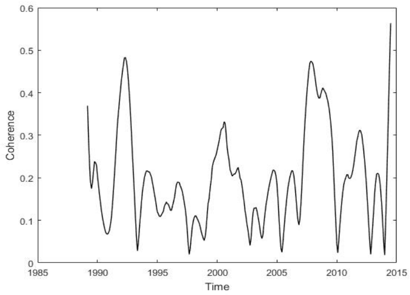
While considering the different patterns, we note that the long-run coherence function is volatile across the study period, with a relatively smooth pattern for euro area and China. A lower average correlation is observed for the UK and is higher for China. Specifically, for the UK, we observe a breakdown of the dynamic interdependence between inflation and IU in 1992, when the inflation targeting policy was adopted. This dramatic decrease is explained by the fundamentals of the inflation targeting policy, characterized by a high degree of commitment and transparency, and a low IU level. Since 1992, the correlation level has remained low and only increased at the beginning of 2000, reflecting the increasing housing prices and leading to a high interest rate and high taxes on house purchases.

Regarding the US, the highest levels of the coherence function reflect the price levels during certain periods of the 1960s (Vietnam War), of 1973 and 1979 (oil shocks), of the end of the 1980s, of 2000 (dot-com bubble), of 2007–2008 (subprime and financial crisis), and of 2010. In the euro area, the coherence function reached high levels after the subprime crisis and during the sovereign debt crisis. In China, the relationship reached high levels during the dot-com bubble, reflecting the absence of an explicit inflation target. In South Africa, the dependence between inflation and IU was low and stable from 1960 to the end of the 1990s. However, at the beginning of 2000, the dynamic interaction between the series reached high levels.

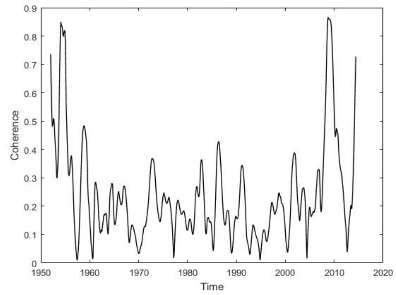
Overall, the nonparametric analysis of the relationship between inflation and IU provides the following results. It identifies the linkage dynamics by sub-period, determines the association with monetary regime policy, captures different stylized facts, and yields an analysis of this relationship. It also shows that the inflation–IU relationship might significantly vary across countries, reflecting the effects of central bank policies, their targeting rules, and degrees of interdependence.

**Figure 3. Long- and short-run coherence function between inflation and IU**

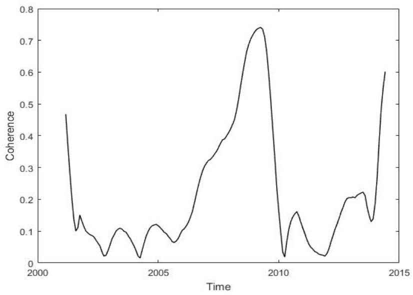
5(a) UK



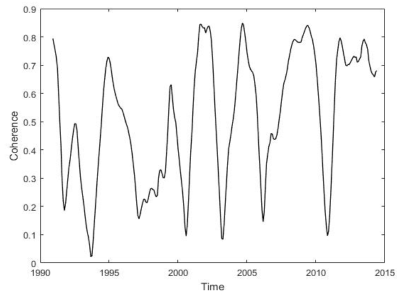

5(b) US



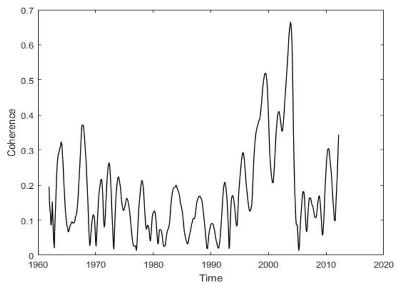

5(c) Euro area




5(d) China




5(e) South Africa

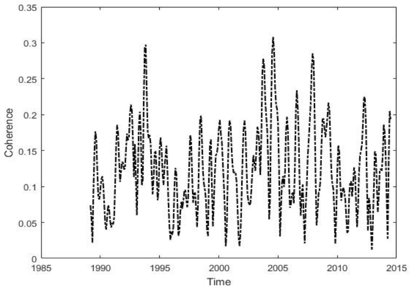



The interdependence between inflation and IU is significantly more noisy and more volatile in the short and medium terms than in the long term. Finally, while the estimate of the short-run coherence function enables capturing the intensity of the interaction between inflation and uncertainty, the interaction dynamic is high during turmoil periods (around 30–50%) and relatively low during stable periods (10–20%).

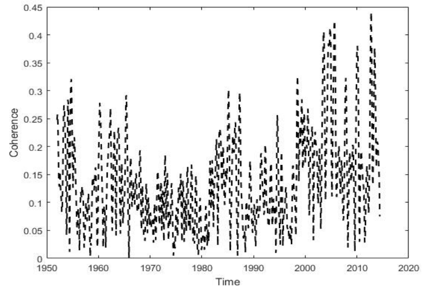
In summary, the evolutionary co-spectral analysis points to a significant relationship between inflation and IU, more pronounced in periods of turmoil than in stable periods. This relationship exhibits significant time variation associated with the market state. The intensity of the relationship increases significantly during turmoil periods (30–50%), while it is lower (10–20%) during calm periods over the short run. This finding confirms the usefulness of our spectral approach, which demonstrates that uncertainty effects can vary across horizons. Investigating this relationship for different horizons can assist policymakers in limiting the uncertainty effects for each horizon.

**Figure 4. Medium-run coherence function between inflation and IU**

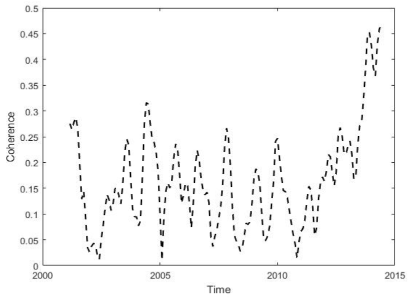
4(a) UK



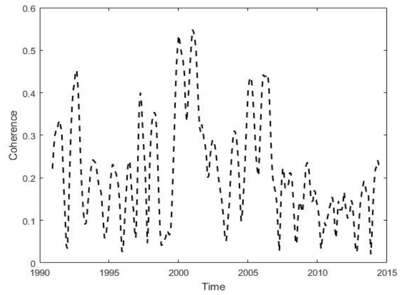

4(b) US



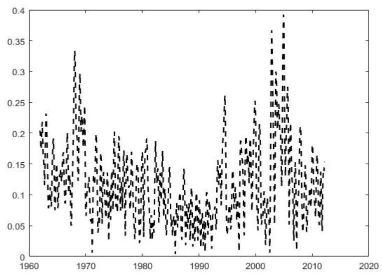

4(c) Euro area




4(d) China



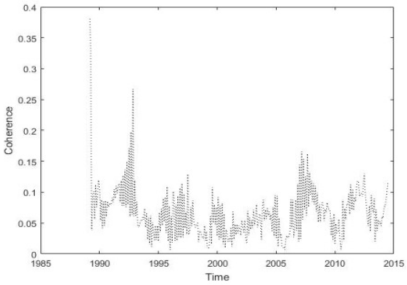

4(e) South Africa



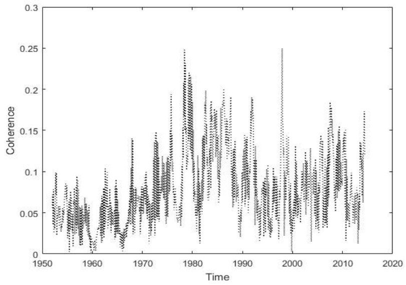
The evolutionary co-spectral analysis contributes to the literature by clarifying the analyzed relationship. Not only it does enable identifying periods with high and low interactions, but also permits measuring the effects of policymakers' actions, such as explicit inflation targeting and degree of central bank independence. However, evolutionary co-spectral analysis does not provide the statistical significance of these relationships, only the estimated magnitude of interdependence. To conduct statistical significance tests, the wavelet approach is used.

**Figure 5. Short-run coherence function between inflation and IU**

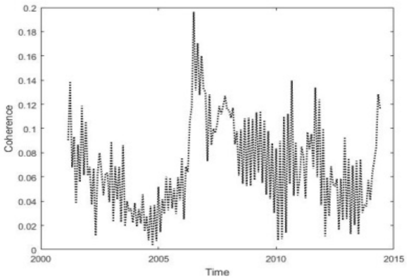
3(a) UK



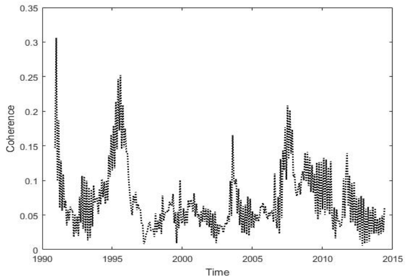

3(b) US



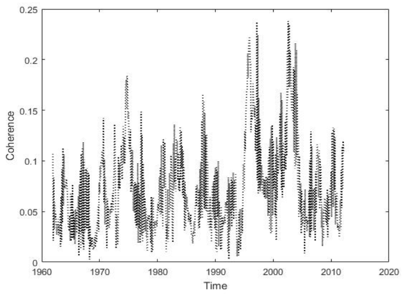

3(c) Euro area




3(d) China




3(e) South Africa




### 4.4.2. Wavelet Analysis

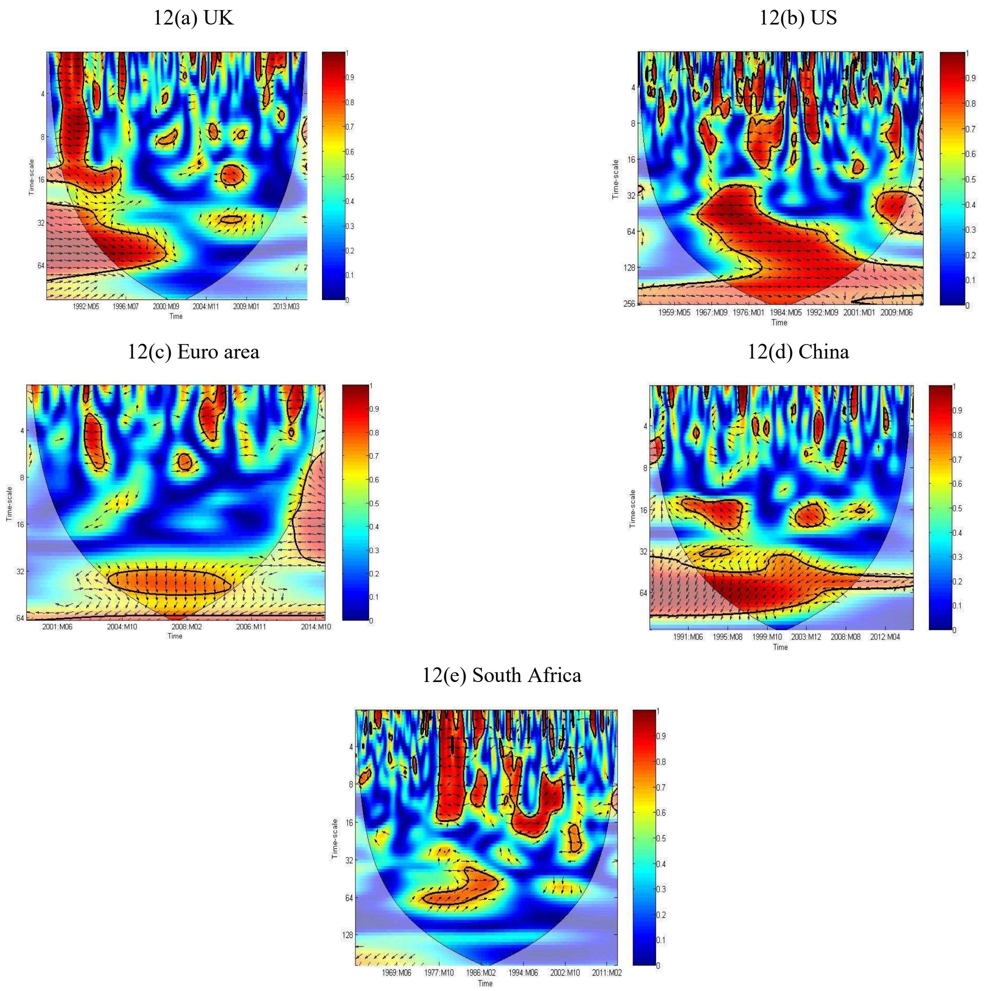
In Figure 6, we report the results of the coherence wavelet function between inflation and IU. The vertical and horizontal axes show the time and frequency dimensions, respectively, while the colors depicts the level of interdependence between 0 and 1. A lower interdependence is indicated in dark blue and a higher correlation is represented in dark red. The left vertical color axis provides more precise information about the interdependence level. The main contribution of the wavelet approach is to provide the causality sign between inflation and IU.

The arrows pointing to the right imply that variables are in a pro-cyclical phase. Specifically, when arrows point right and up, they imply IU is lagging, while when they point right and down, they imply IU is leading. When the arrows point to the left, the series are counter-cyclical. When they point left and down, IU is lagging, and when they point left and up, IU is leading.

The wavelet coherence function analysis provides two clear conclusions. First, we confirm the analysis results of the evolutionary co-spectral analysis in terms of the significant time-variation of co-movements between inflation and IU. The co-movements vary by country and are more pronounced in the short term than the medium and long terms. Figure 6(a) displays a high interdependence between the UK inflation rate and its uncertainty at the beginning of the 1990s over the short and medium horizons (0–16 scales). For the short horizon (0–4 scales), the interdependence between UK inflation and IU is high in 1997, during 2000–2001 (internet bubble), and during 2008–2009 (subprime crisis). For the medium term, the relationship between inflation and IU is important during two episodes: the period of the "great moderation" (1990s) and the subprime crisis (2008–2009). In the long term, the relationship between inflation and IU is observed only during the 1990s.

For the US, dependence is observed in the short, medium, and long horizons during periods of turmoil, including the late 1960s, the 1970s and 1980s, and the beginning of the 1990s in the short and medium run. Since the latter period, interdependence is only observed over the short and medium horizons, such as during the subprime crisis. For the euro area, the relationship is observed in the short term (0–8 scales) during 2001–2002 (the internet bubble), 2007–2008 (the subprime crisis), and 2011–2012 (European sovereign debt problem). Furthermore, a long run relationship is observed in the 2004–2008 period. Finally, a similar behavior is seen for South Africa, where interdependence is more pronounced during periods of turmoil. Particularly, the relationship between inflation and IU is tighter during the 1970s over the short and medium terms (0–16 scales). This behavior has been reproduced during 1985–1988 for both the short and medium terms. A high interdependence is also observed since the beginning of 2000 over the medium horizon.

**Figure 6. Coherence between inflation rate and IU**




Wavelet analysis confirms the findings of the evolutionary co-spectral approach and shows a significant relationship that is more noisy and more volatile over the medium and short terms than the long term. The intensity of the relationship increases significantly during turmoil periods, while a long-term relationship is observed only during crisis periods. This finding is consistent with Evans' (1991) decomposition of IU into short run and long run. He suggests that agents' temporal decisions are more likely sensitive to the conditional variance of short-run inflation movements. However, intertemporal decisions are more likely dependent to the changes in the conditional variance of long-term inflation. Indeed, we have identified a strong relationship between inflation and IU in the short term, which can be explained by the agent's temporal decisions during a stable period, such as their inflation forecasting regarding the state of the economy. We also identified a long-run relationship during a period of crisis that can be explained by intertemporal decisions, such as a change of monetary policy in response to crises or structural change to overcome economic instability (see, e.g., Caporale and Kontonikas, 2009).

Additionally, the wavelet approach produces different directions of the arrows, suggesting that the relationship exists, but alternates between positive and negative according to the economic state and monetary policy. In some cases, the arrows point to the left, implying that inflation and uncertainty may be counter-cyclical, such as in the euro area during 2001–2002, 2007–2008, and 2011–2012 over the short term; the US 2008–2009 over the medium term; and China during 1990–1995 over the long term. In other cases, the arrows point to the right, implying a pro-cyclical relationship, such as in the UK during 1990–1995 over the short and medium runs, in 1997 over the short run and during 2009-2012 over the short run. For the US, the arrows point to the right during all periods of high interdependency described above, except the subprime crisis, where they point to the left. For South Africa, the arrows point to the right for all periods for scales between 0 and 32. These findings confirm the stylized facts observed in Figure 1, when we highlight similar movements in the same direction between series in some cases and in opposite directions in others. Additionally, our results show that causality varies across frequencies. In some cases, inflation is lagging and, in others, it is leading.

For all studied countries, we observe that for short-term frequencies, the arrows point to the left and down, right and up, or right and down, implying counter-cyclical or pro-cyclical relationships, with IU leading or lagging. These findings support the theoretical hypotheses summarized in Table 1. In short, the relationship between inflation and IU might be positive or negative, and the causality alternates across frequencies and countries. For example, for the UK over the short run, the arrow is pointing to the right and down during stable periods. At the beginning of the 1990, the arrow points to the right, implying a pro-cyclical positive relationship between inflation and IU, with IU leading over the short and medium terms. During the subprime crisis, the arrows point to the right and down for medium-term frequency and right and up for long-term frequency, implying a pro-cyclical positive relationship between inflation and IU. Causality alternates between horizons. These results support the Friedman–Ball and Cukierman and Meltzer (1986) hypotheses. The change of causality in the long-term frequency after the subprime crisis might be explained by the nonconventional policy adopted with the objective to reduce uncertainty and ensure economic stability. As this period is characterized by a high degree of economic uncertainty, the IU could not continue to predict inflation.

However, the pattern of the relationship between inflation and IU is different for the US. Over the short term, the arrows point to the right and up from the 1950s to the end of the 1990s. This result reflects a pro-cyclical relationship, with inflation causing the IU. Since the 2000s, the relationship is only observed during periods of turmoil (2001 and the subprime crisis). During these periods, the arrows point to the left and down, implying that the relationship is countercyclical, with IU causing inflation. For the euro area and China, the arrows usually point to the left, and causality alternates between IU lagging and leading. For South Africa, the arrows point to the right and up for all time-scales, implying a pro-cyclical relationship, with inflation causing IU.

### 4.5.Modeling the Relationship between IU and Output growth

In this part, we aim to complete the investigation of Friedman hypothesis through interesting on the IU-output growth relationship. Based on data availability, we study this relationship for the Euro area, UK, and USA. We computed the monthly output growth as the difference of logarithm of the monthly industrial production between two successive months. The data are collected for Federal reserve bank database and ranged from 1999:01 to 2015:03, 1988:01 to 2015:03, and 1950:01 to 2015:03 for Euro area, UK, and USA, respectively.

**Figure 7. The coherence function between IU-Output growth for the Euro Area**

Figure 7.a. The LT coherence function

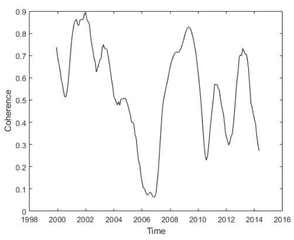



Figure 7.b. The MT coherence function

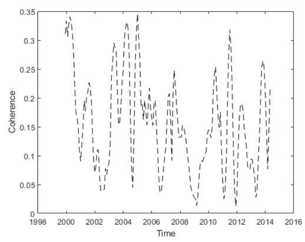



Figure 7.c. The LR coherence function

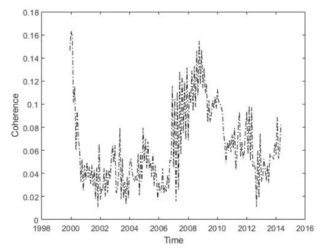



**Figure 8. The coherence function between IU-Output growth for the UK**

Figure 8.a. The LT coherence function

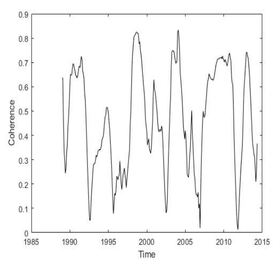



Figure 8.b. The MT coherence function

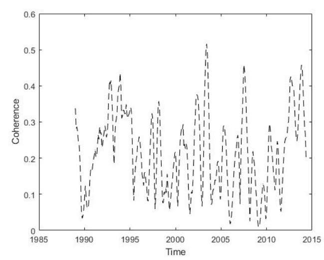



Figure 8.c. The LR coherence function

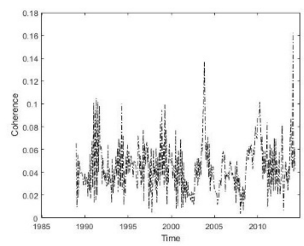



**Figure 9 The coherence function between IU-Output growth for the USA**

Figure 9.a. The LT coherence function

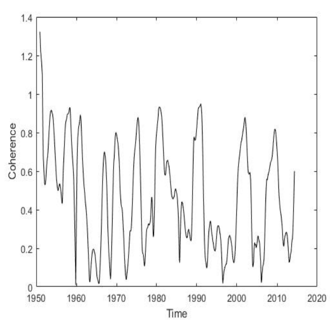



Figure 9.b. The MT coherence function

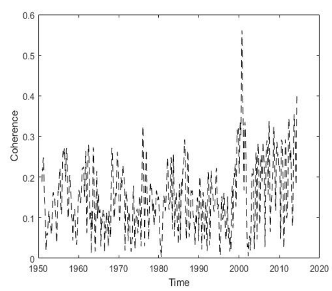
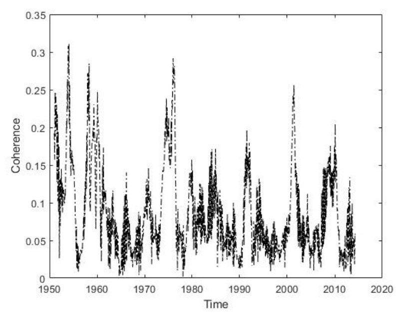


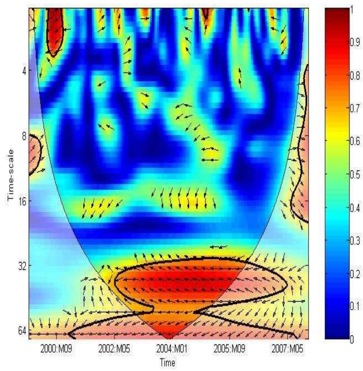
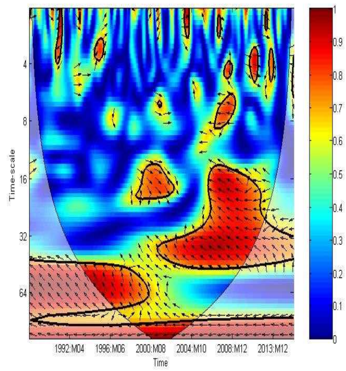
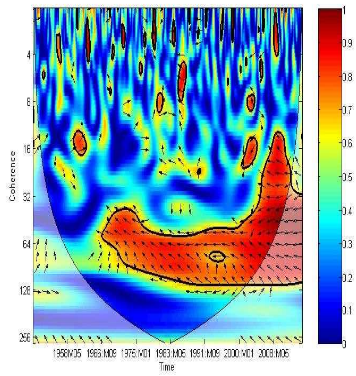

Figure 9.c. The LR coherence function




Note: LT, MT, and ST denote the long term, medium term and short term.

Figs 7, 8, and 9 show some worthy aspects. First, we show a high time-varying interaction between IU and output growth in long-run horizon (Figs.7a, 8a, and 9a). For the case of Euro area the LT the dynamic correlation is higher than 60% in average. Interestingly, we show that the relationship decreases intensely during period of crisis, such in 2001, 2008, 2010 for internet bubble, subprime crisis and European sovereign debt, respectively. For the medium term the interaction between, the relationship between IU and output growth disappear, as in average is around 10-20% (Figs, 7b, 8b, and 9b). More specifically, there is no relationship in short term dynamic (Figs 7c, 8c, and 9c).

Figure 10. The coherence between IU-Output growth

| Figure 10.a. The coherence function for the Euro area | Figure 10.b. The coherence function for the UK | Figure 10.c. The coherence function for the USA |
|---|---|---|
|  |  |  |

The wavelet approach results support the evolutive spectral approach results, (Fig 10). For all countries, we show that there no correlation for short-term and medium term, as less than 32 time-scales (approximate less than 3 years). For long term time scale, we observe significant relationship more than 70%.

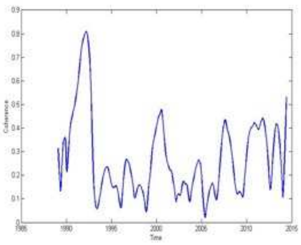
**4.6. Robustness Tests**

The robustness check concerns the coherence of the wavelet approach. We rely on the theoretical distribution defined by Torrence and Compo (1998), as per Equation (12). However, this distribution has been criticized by Liu et al. (2007) and Veleda et al. (2012) in terms of low-frequency oscillations, leading to ambiguity in the wavelet power spectrum estimation. For robustness confirmation and to avoid bias, we re-estimate the cross-wavelet spectrum, as defined by Ng and Chan (2012). The results show significant similarities between the two different estimation methods.[20] This confirms the robustness of our findings and conclusions.

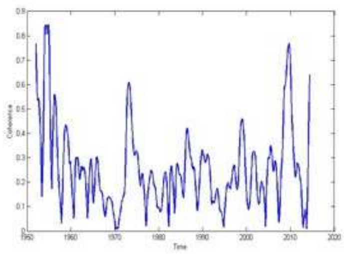
We check the robustness of our results using other measures of IU. More specifically, we check the sensitivity of our results through using parametric approaches based GARCH family models. Interestingly, we employ symmetric and asymmetric version of GARCH family models based on GARCH(1,1) and EGARCH(1,1) models. Figs. 11, 12 and 13 present the coherence function between symmetric IU measure and inflation for long-run, medium-run, and long-run in order to take into account different stylized facts of volatility series[21].

**Figure 11.** Long-run Coherence Function between Inflation and Uncertainty

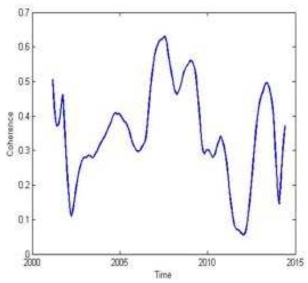
11(a) UK



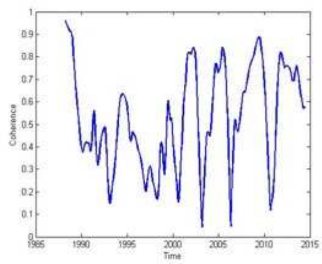
11(b) US



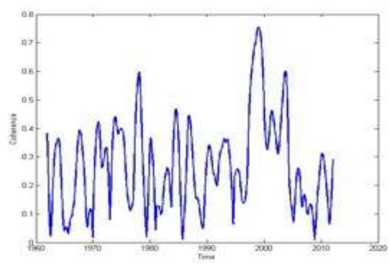
11(c) Euro area



11(d) China



11(e) South Africa



[20] Due to space considerations, we have not reported the graphs for these results. However, they are available upon request.
[21] Hereafter, we report only the results based on GARCH(1,1) estimation. The results of E-GARCH (1,1) are not reported to save space, but available upon request.

**Fig.12. The medium-run coherence function between inflation and its uncertainty**

Fig. 12(a). The UK

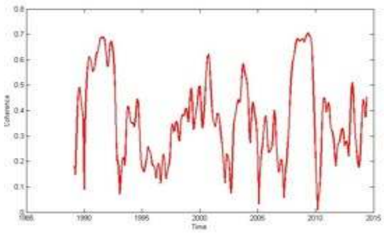


Fig. 12(b). The US

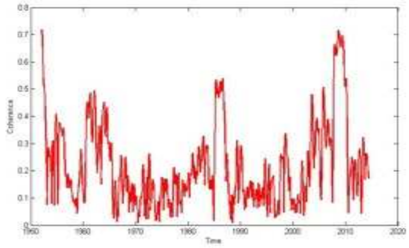


Fig. 12(c). Euro Area

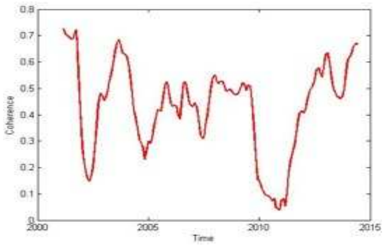


Fig. 12(d). China

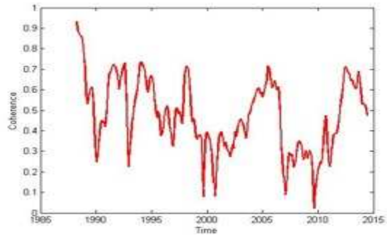


Fig. 12(e). South Africa

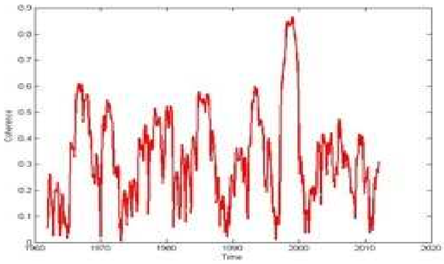
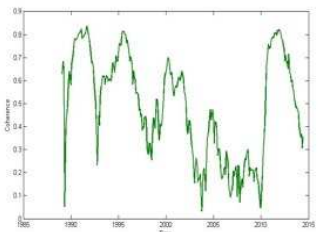
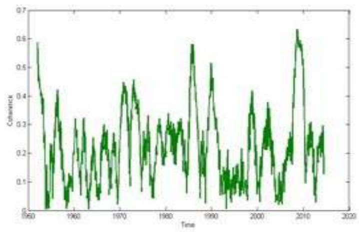


**Figure 13.** Short-run Coherence Function between Inflation and Uncertainty

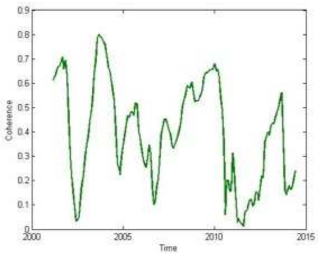
13(a) UK



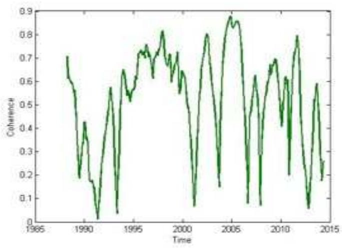
13(b) US



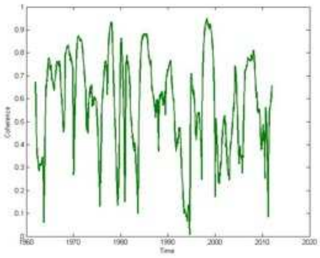
13(c) Euro area



13(d) China



13(e) South Africa



Figs. 11, 12 and 13 exhibit similar patterns with their corresponding Figs. 3, 4 and 5 based on the stochastic volatility. Similar to the spectral approach, our results of the relationship between IU measured through GARCH model and the inflation based on the wavelet methodology are robust (Fig.14).

Figure 14. The coherence between inflation rate and its uncertainty

**Fig.14 (a).** The UK

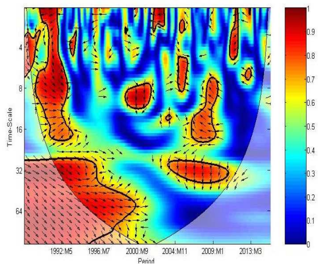


**Fig.14 (b) The US**

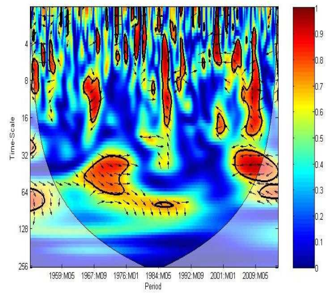


**Fig.14 (c).** Euro Area

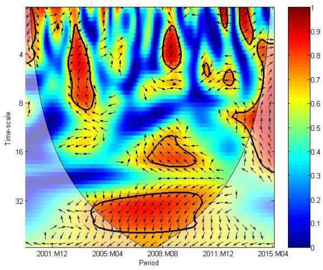


**Fig.14 (d).** China

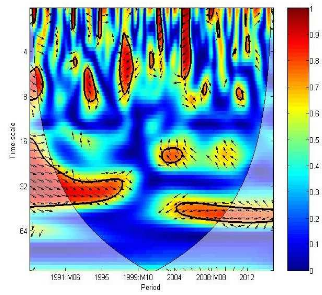
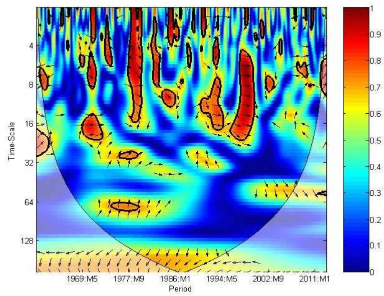


**Fig.14 (e).** South Africa



The results of the relationship between inflation and IU based on EGARCH models is similar to those with GARCH and with stochastic volatility model. Finally, the results regarding the relationship IU-output growth are robust.[22]

## 5. Conclusion

The measurement of IU is a crucial topic for both policymakers and economic agents. For policymakers, a relevant uncertainty measure leads to the adoption of appropriate monetary policy actions, which is more active for high uncertainty and less so otherwise. For economic agents, higher uncertainty leads to more frequent negotiations of nominal contracts. This study analyzes the relationship between inflation and IU for five major countries and regions (the US, the UK, the euro area, China, and South Africa). The topic is investigated during crises and downturn periods in economies with explicit versus implicit inflation targets, conventional versus unconventional monetary policies, and independent versus dependent monetary policies.

Our paper estimates IU using the stochastic volatility model. We also propose a novel econometric specification for the inflation–IU relationship. Our findings are as follows. First, there is a significant relationship between inflation and uncertainty, which exhibits time-variation and changes across frequencies and time. Indeed, the relationships are more significant in the short term than in the long term. Additionally, this relationship seems to increase during periods of crises and downturns. Second, this relationship alternates between being positive for stable periods, where IU is lagging—thereby confirming the Friedman theory—and negative during crises. Finally, the significant differences between countries highlight the effects of monetary regimes on uncertainty and could be helpful in selecting appropriate and timely monetary policies to limit uncertainty effects.

Our results contribute to the literature in various ways. The results address the

[22] We ddo report the results of this second relationship of Friedman hypothesis in order to save place, but results are available upon  request.

contradictions in the literature, particularly by resolving the complexity and ambiguity noted in previous related studies on the inflation–IU relationship. Specifically, our results highlight that the relationship between inflation and IU can be positive (the Friedman–Ball hypothesis) or negative (the Holland hypothesis), depending on whether the economic environment is stable or turbulent. Our findings also show the trend of the relationship across frequency and time. We provide insights relevant to reducing or minimizing the marginal effects of inflation on IU. In different contexts—such as calm or turbulent periods and price stability monetary objectives—this relationship depends on the monetary policy under. We show that uncertainty is lower when a price stability objective exists.

Previous studies have identified varying and seemingly conflicting relationships between inflation and IU. Our more general approach includes the prior results into a unified framework, clarifying the circumstances of the ambiguity in this relationship and identifying the causes for each case.

## References


Albulescu, C.T., Tiwari, A.K., Miller, S.M., Gupta, R. (2019). Time–frequency relationship between US inflation and inflation uncertainty: evidence from historical data. Scottish Journal of Political Economy 66(5), 673-702.

Allégret, J.P., Essaadi, E., 2011. Business cycles synchronization in East Asian economy: Evidences from time-varying coherence study. Economic. Modelling. 28, 351–365.

Artis, M.J., Bladen-Hovell, R., Nachane, D.M., 1992. Instability of the velocity of money: A New approach based on the evolutionary spectrum. CEPR Discussion Paper 735.

Baillie, R.T., Chung, C., Tieslau, M.A., 1996. Analysing inflation by the fractionally integrated ARFIMA–GARCH model. Journal of. Applied Econometrics. 11, 23–40.

Ball, L., 1992. Why does high inflation raise inflation-uncertainty? Journal of Monetary Economics. 29, 371–388.

Barnett, W.A., Chen, G., 2015. Bifurcation of macroeconometric models and robustness of dynamical inferences. Found. Trend Econom. 8 (1–2), 1–144.

Barnett, W.A., Serletis, A. and Serletis, D., 2015. Nonlinear and complex dynamics in economics. Macroeconomic Dynamics. 19, 1749–1779.

Barnett, W.A., Gallant, A.R., Hinich, J.A., Jungeilges, D.T., Kaplan, D.T., and Jensen, M.J., 1997. A single-blind controlled competition among tests for nonlinearity and chaos. Journak of Econometrics. 82, 157–192.

Barro, R.J., Gordon, D.B., 1983. Rules, discretion and reputation in a model of monetary policy. Journal of Monetary Econonomics. 12, 101–121.

Ben Nasr, A., Balcilar, M., Ajmi, A.N., Aye, G.C., Gupta, R., Eyden, R., 2015. Causality between inflation and inflation uncertainty in South Africa: Evidence from a Markov-switching vector autoregressive model. Emerging Market Review. 24, 46–68.

Bernanke, B.S., Mishkin, F.S., 1992. Central bank behavior and the strategy of monetary policy: Observations from six industrialized countries. NBER Working Paper 4082.

Berument, H., Yalcin, Y., Yildirim, J., 2009. The effect of inflation uncertainty on inflation: Stochastic volatility in mean model within a dynamic framework. Economic Modelling. 26, 1201–1207.

Bloomfield, D.S., McAteer, R.T.J., Lites, B.W., Judge, P.G., Mathioudakis, M., Keenan, F.P., 2004. Wavelet phase coherence analysis: Application to a quiet-sun magnetic element. Astrophys. J. 617, 623–632.

Canzoneri, M., 1985. Monetary policy games and the role of private information. American Economic Revieew. 75. 1056–1070.

Caporale, G.M., Kontonikas, A., 2009. The Euro and inflation uncertainty in the European Monetary Union. Journal of Intlional Money and Finance. 28. 954–971.

Carnero, M. A., Pena, D., Ruiz, E., 2004. Persistence and kurtosis in GARCH and stochastic volatility models. Journal of . Financial. Economics. 2:3, 19–342.

Chan, J., Grant, A., 2016. Modeling energy price dynamics: GARCH versus stochastic volatility. Energy Economics. 54, 182–189.

Chan J.C.C., Hsiao, C.Y.L., 2014. Estimation of stochastic volatility models with heavy tails and serial dependence, in: Jeliazkov, I. Yang, X.-S. (Eds.), Bayesian Inference in the Social Sciences. John Wiley & Sons, Hoboken.

Chan, J.C.C., 2013. Moving average stochastic volatility models with application to inflation forecast. Journal of Econometrics. 176, 162–172.

Chan, J.C.C., 2015. The stochastic volatility in mean model with time-varying parameters: An application to inflation modeling. Journal of Business Economics Statistics. 35, 17–28.

Chang, K., 2012. The impacts of regime-switching structures and fat-tailed characteristics on the relationship between inflation and inflation uncertainty. Journal of Macroeconomics. 34, 523–536.

Conrad, C., Karanasos, M., 2005. On the inflation-uncertainty hypothesis in the USA, Japan and the UK: A dual long memory approach. Jpn. World Econ. 17, 327–343.

Creal, D.D., Wu, J.C., 2014. Interest rate uncertainty and economic fluctuations. Working paper 14-324, The University of Chicago Booth School of Business.

Cukierman, A., 1992. Central Bank Strategy, Credibility, and Independence: Theory and evidence. MIT Press, Cambridge.

Cukierman, A., Meltzer, A.H., 1986. A theory of ambiguity, credibility, and inflation under discretion and asymmetric information. Econometrica. 54, 1099–1128.

Goupillaud, P., Grossman, A., Morlet, J., 1984. Cycle-octave and related transforms in seismic signal analysis. Geoexplor. 23, 85–102.

Emery, K.M., 1993. Inflation and its variability: An alternative specification. Applied Economics. 25, 43–46.

Evans, M., 1991. Discovering the link between inflation rates and inflation uncertainty. Journal of Money Credit Bank. 2, 169–184.

Evans, M., Wachtel, P., 1993. Inflation regimes and the sources of inflation uncertainty. J. Money Credit Bank. 25, 475–511.

Ferreira, D., Palma, A., 2016. Inflation and inflation uncertainty in Latin America: A time-varying stochastic volatility in mean approach. Journal of Economic Studies. 44, 506–517.

Fischer, S., Modigliani, F., 1978. Toward and understanding of the real effects and costs of inflation. Weltwirtschaftliches Archiv. 114, 810–833.

Friedman, M., 1977. Nobel lecture: Inflation and unemployment. Journal of Political Economy. 85, 451–472.

Ftiti, Z., 2010. The macroeconomic performance of the inflation targeting policy: An approach based on the evolutionary co-spectral analysis. Economic Modelling 27, 468–476.

Ftiti, Z., Jawadi, F., 2018. Forecasting inflation uncertainty in the United States and Euro Area. Computional Economics. https://doi.org/10.1007/s10614-018-9794-9

Gallegati, M., Ramsey, J.B., Semmler, W., 2014. Interest rate spreads and output: A time scale decomposition analysis using wavelets. Computional Statistics and Data Anallysis. 76, 283–290.

Golub, J., 1994. Does inflation uncertainty increase with inflation. Federal Reserve Bank of Kansas City, Third Quarter, 28–38.

Gourieroux, C., Sufana, R., 2010. Derivative pricing with Wishart multivariate stochastic

volatility. J. Bus. Econom. Stat. 28, 438–451.
Grier, K.B., Perry, M.J., 1998. On inflation and inflation uncertainty in the G7 countries. Journal of. International Money and Finance. 17, 671–689.

Grinsted, A., Moore, J., Jevrejeva, S., 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. Nonlin. Proc. Geophys. 11, 561–566.

Haven, E., Liu, X., Shen, L., 2012. De-noising option prices with the wavelet method. European Journal of Operational Research 222, 104–112.

Holland, S., 1993. Comment on inflation regimes and the sources of inflation uncertainty. Journal of Money Credit and Banking. 25, 514–520.

Holland, S., 1995. Inflation and uncertainty: Tests for temporal ordering. Journal of Money Credit and Banking. 27, 827–837.

Hou, C. 2019. Time-Varying Relationship between Inflation and Inflation Uncertainty. Oxford Bulletin of Economics and Statistics 82(1), 83-124

Jensen, M.J., 2004. Semiparametric Bayesian inference of long-memory stochastic volatility models. Jourlan of Time Series Analysis 25 (6), 895–922.

Jensen, M.J., 2009. The long-run Fisher effect: can it be tested. Journal of  Money Credit and Banking. 41, 221–231.

Johnson, D.R., 2002. The effect of inflation targeting on the behavior of expected inflation: Evidence from an 11 country panel. Journal of Monetary Economics. 49, 1521–1538.

Kontonikas, A., 2005. Inflation and inflation uncertainty in the United Kingdom, evidence from GARCH modelling. Economic Modelling. 21, 525–543.

Koopman, S.J., Mallee, M.I., Van der Wel, M., 2010. Analyzing the term structure of interest rates using the dynamic Nelson–Siegel model with time-varying parameters. Journal of Business Economics and Statistics. 28, 329–343.

Liu, Y., Liang, X.S., Weisberg, R.H., 2007. Rectification of the bias in the wavelet power spectrum. J. Atmos. Ocean. Technol. 24, 2093–2102.

Logue, D., Willett, T., 1976. A note on the relation between the rate and variability of inflation. Economica. 43, 151–158.

Madaleno, M., Pinho, C., 2014. Wavelet dynamics for oil-stock world interactions. Energy Economics. 45, 120–133.

Mallick, S, Sousa, R., 2013. Commodity prices, inflationary pressures, and monetary policy: Evidence from BRICS economies. Open Economic Review. 24, 677–694.

Mallick, S., Mohsin, M., 2016. Macroeconomic effects of inflationary shocks with durable and non-durable consumption. Open Economic. Review. 27, 895–921.

Neanidis, K.C., Savva, C.S., 2011. Nominal uncertainty and inflation: The role of European Union membership. Economics Letters. 112, 26–30.

Ng, E.K., Chan, J.C., 2012. Geophysical applications of partial wavelet coherence and multiple wavelet coherence. J. Atmos. Ocean. Technol. 29, 1845–1853.

Pourgerami, A., Maskus, K.E., 1987. The effects of inflation on the predictability of price changes in Latin America: Some estimates and policy implications. World Development. 15, 287–290.

Priestley, M.B., 1965. Evolutionary spectra for non-stationary processes. Journal of Royal Statistics Society. B 27, 204–237.

Priestley, M.B., 1966. Design relations for non-stationary processes Journal of Royal Statistics Society. B 28, 228–240.

Priestley, M.B., 1988. Non-Linear and Non-Stationary Time Series Analysis. Academic Press, London.

Priestley, M.B., Tong, H., 1973. On the analysis of bi-variate non-stationary processes. Journal of Royal Statistics Society. B 35, 135–166.

Rua, A., Nunes, L.C., 2012. A wavelet-based assessment of market risk: The emerging markets case. Quarterly Review in Economics and Finance 52, 84–92.

Rossi, B., Sekhposyan, T., Soupre, M., 2016. Understanding the sources of macroeconomic uncertainty. Working Papers 920, Barcelona Graduate School of Economics.

Serletis, A., Rahman, S., 2009a. The output effects of money growth uncertainty: Evidence from a multivariate GARCH-in-Mean VAR. Open Economic Review 20, 607–630.

Serletis, A., Rahman, S., 2009b. On the output effects of monetary variability. Open Economic Review. 26, 225–236.

Serletis, A., Shahmoradi, A., 2006. Velocity and variability of money growth: Evidence from a VARMA, GARCH-M model. Macroeconomic Dynamics. 10, 652–666.

Serletis, A., Xu, L., 2017. Money supply volatility and the macroeconomy. University of Calgary, Working paper.

Torrence, C., Compo, G.P., 1998. A practical guide to wavelet analysis, Bull. Am. Meteorol. Soc. 79, 605–618.

Torrence, C., Webster, P., 1999. Interdecadal changes in the ENSO-monsoon system. Journal of Climate. 12, 2679–2690.

Ungar, M., Zilberfarb, B., 1993. Inflation and its unpredictability—Theory and empirical evidence. Journal of Money Credit and Banking 25, 709–720.

Van Bellegem, S., 2013. Locally stationary volatility modelling, in: Bauwens, L., Hafner, C., Laurent, S. (Eds.), Handbook in Financial Engineering and Econometrics: Volatility models and their applications. Wiley, New York, Chapter 10.

Van Bellegem, S., Von Sachs, R., 2008. Locally adaptive estimation of evolutionary wavelet spectra. Journal of Statistics. 36, 1879–1924.

Veleda, D., Montagne, R., Araujo, M., 2012. Cross-wavelet bias corrected by normalizing scales. J. Atmos. Ocean. Technol. 29, 1401–1408.

Zapodeanu, D., Cociuba, M-L., Petris, S., 2014. Inflation uncertainty and inflation in the case of Romania, Czech Republic, Hungary, Poland and Turkey. Proceedings in Economics and Financ. 15, 1225–1234.

Zar, J.H., 1999. Biostatistical Analysis, fourth ed. Prentice Hill.